

# MODELING FUZZY DECISION TREES FROM FUZZY SETS<sup>\*</sup>

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## Introduction

Current electronic data repositories contain enormous amounts of data. These data include also currently unknown and potentially interesting patterns and relations that can be uncovered using knowledge discovery and data mining methods [1]. Commonly supervised machine learning is used, in which there exists a set of training instances represented by a vector of the values of features and a class label.

In this paper we are present our approach that can deal with fuzzy defined data. These data are more accuracy to reflect the real around world. We use a technique to compute *cumulative* information estimations [2]. The use of such estimations allows inducing minimum cost fuzzy decision trees (FDT) based on different criteria of optimality. As example we will introduce three types in this paper: *unordered*, *ordered* and *reliability* FDT.

## 1. Fuzzy Decision Trees

Quinlan proposed an ID3 algorithm to design crisp decision trees [3]. The first idea of FDT was introduced by Chang and Pavlidis [4]. They presented FDT structure and a search method and described a difference between crisp and FDT. The generalizing ID3 algorithm for fuzzy sets was resulted into Fuzzy ID3 algorithm and its known variants that can be found in [2, 4-11]. The classification task in [5] has nominal or numerical values as input and output attributes. No fuzziness was involved with nominal data. Kosko's fuzzy entropy was used to measure the fuzziness of classification by the neuron [6]. Yuan and Shaw proposed construction a FDT in the process of reducing classification *ambiguity* with accumulated fuzzy *evidences* [7]. This algorithm was developed in [8].

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## 2. Fuzzy Decision Trees Induction

A typical classification problem can be described as follows [7-8]. A universe of objects  $U=\{u\}$  is described by  $N$  training examples and  $n$  input attributes  $A=\{A_1, \dots, A_n\}$ . Each attribute  $A_i$  ( $1 \leq i \leq n$ ) measures some important feature and is presented by a group of discrete *linguistic terms*. We assume that each group is a set of  $m_i$  ( $m_i \geq 2$ ) values of fuzzy subsets  $\{A_{i,1}, \dots, A_{i,j}, \dots, A_{i,m_i}\}$ .

The cost of an attribute  $A_i$  denoted as  $Cost_i$  is an integrated measure that accounts financial and temporal costs that are required to define the value of the  $A_i$  for a certain subject. We will suggest that each object  $u$  in the universe is classified by a set of classes  $\{B_1, \dots, B_{m_b}\}$ . This set describes by output attribute  $B$ .

In this paper we introduce an approach for a sequence of expanded attributes testing, i.e. determination of input attributes' values  $\{A_{1i}, \dots, A_{ni}\}$  of a new subject, that allows to accomplish the correct diagnostics. It is obvious that the problem is that the quasioptimal sequence should guarantee as correct diagnostics with a priori defined level of accuracy as minimum cost for accomplishing the tests and procedures.

*Example 1.* (adopted from [7-8] with some modifications) An object is presented with four attributes:  $A=\{A_1, A_2, A_3, A_4\}$  and one output attribute  $B$ . Each attribute has values:  $A_1=\{A_{1,1}, A_{1,2}, A_{1,3}\}$ ,  $A_2=\{A_{2,1}, A_{2,2}, A_{2,3}\}$ ,  $A_3=\{A_{3,1}, A_{3,2}\}$ ,  $A_4=\{A_{4,1}, A_{4,2}\}$  and  $B=\{B_1, B_2, B_3\}$ .

The membership and cardinality measure of values of these attributes is presented in Table.

We propose new interpretation of Fuzzy ID3, which is based on *cumulative* information estimates [2] for data presented in Table. We realize three types of FDT with different properties: (a) *unordered FDT* (see Fig.1); (b) *ordered FDT*, in which to every node of one level associate similar expanded attribute (see Fig.2); (c) *reliability FDT* which more stable for different noise (see Fig.3).

*Table.* A small training set

No	A <sub>1</sub>			A <sub>2</sub>			A <sub>3</sub>		A <sub>4</sub>		B		
	A <sub>11</sub>	A <sub>12</sub>	A <sub>13</sub>	A <sub>21</sub>	A <sub>22</sub>	A <sub>23</sub>	A <sub>31</sub>	A <sub>32</sub>	A <sub>41</sub>	A <sub>42</sub>	B <sub>1</sub>	B <sub>2</sub>	B <sub>3</sub>
Cost <sub>i</sub>	0,2			0,15			0,45		0,2				
1.	0,9	0,1	0,0	1,0	0,0	0,0	0,8	0,2	0,4	0,6	0,0	0,8	0,2
2.	0,8	0,2	0,0	0,6	0,4	0,0	0,0	1,0	0,0	1,0	0,6	0,4	0,0
3.	0,0	0,7	0,3	0,8	0,2	0,0	0,1	0,9	0,2	0,8	0,3	0,6	0,1
4.	0,2	0,7	0,1	0,3	0,7	0,0	0,2	0,8	0,3	0,7	0,9	0,1	0,0
5.	0,0	0,1	0,9	0,7	0,3	0,0	0,5	0,5	0,5	0,5	0,0	0,0	1,0
6.	0,0	0,7	0,3	0,0	0,3	0,7	0,7	0,3	0,4	0,6	0,2	0,0	0,8
7.	0,0	0,3	0,7	0,0	0,0	1,0	0,0	1,0	0,1	0,9	0,0	0,0	1,0
8.	0,0	1,0	0,0	0,0	0,2	0,8	0,2	0,8	0,0	1,0	0,7	0,0	0,3
9.	1,0	0,0	0,0	1,0	0,0	0,0	0,6	0,4	0,7	0,3	0,2	0,8	0,0
10.	0,9	0,1	0,0	0,0	0,3	0,7	0,0	1,0	0,9	0,1	0,0	0,3	0,7
11.	0,7	0,3	0,0	1,0	0,0	0,0	1,0	0,0	0,2	0,8	0,3	0,7	0,0
12.	0,2	0,6	0,2	0,0	1,0	0,0	0,3	0,7	0,3	0,7	0,7	0,2	0,1
13.	0,9	0,1	0,0	0,2	0,8	0,0	0,1	0,9	1,0	0,0	0,0	0,0	1,0
14.	0,0	0,9	0,1	0,0	0,9	0,1	0,1	0,9	0,7	0,3	0,0	0,0	1,0
15.	0,0	0,0	1,0	0,0	0,0	1,0	1,0	0,0	0,8	0,2	0,0	0,0	1,0
16.	1,0	0,0	0,0	0,5	0,5	0,0	0,0	1,0	0,0	1,0	0,5	0,5	0,0
M(A <sub>i</sub> )	6,6	5,8	3,6	6,1	5,6	4,3	5,6	10,4	6,5	9,5	4,4	4,4	7,2

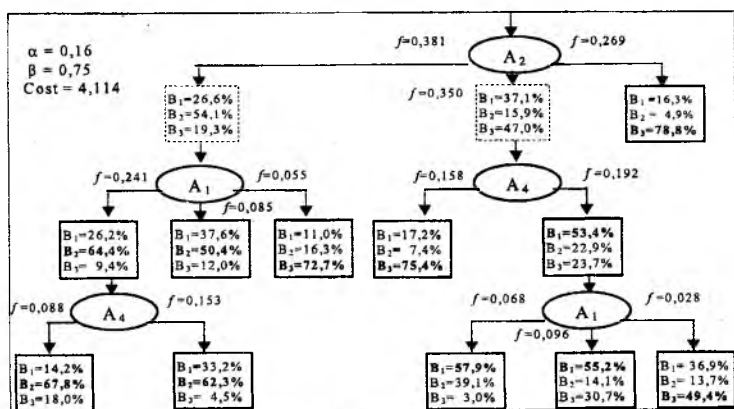


Figure 1. Unordered FDT

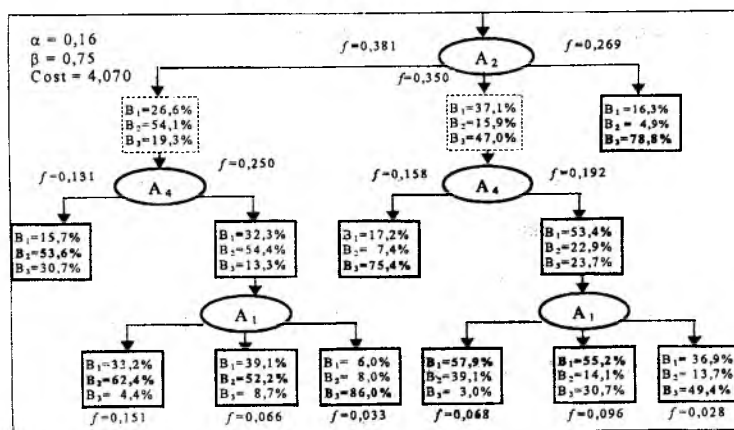


Figure 2. Ordered FDT

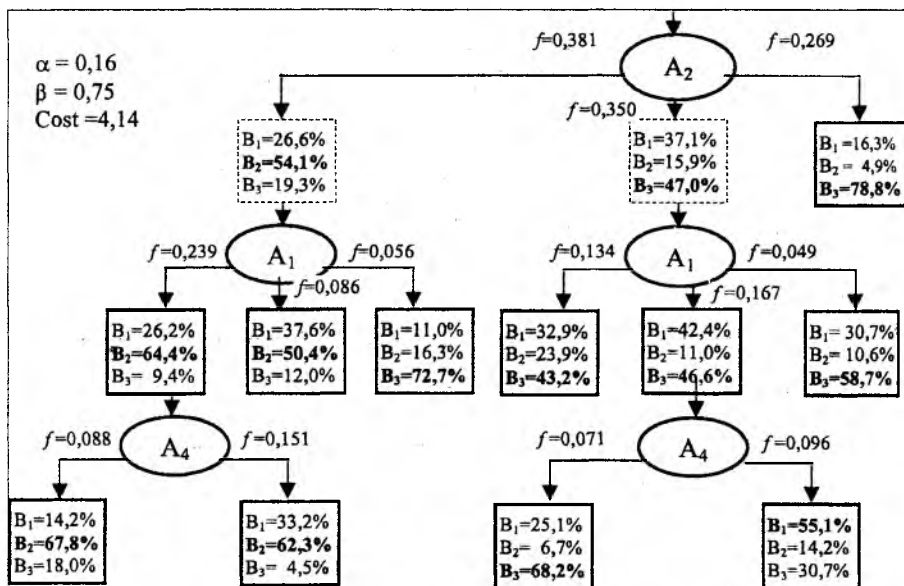


Figure 3. Reliability FDT

#### 4. Conclusion

In this paper we have shown an application of our information estimations [2] in a modeling "greedy" Fuzzy ID3 algorithm for induction of FDT. The use of such estimates allows inducing minimum cost FDT based on different criteria of optimality. We calculated the cost of FDT into considered algorithms.

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