

## ЗАРУБЕЖНЫЙ ОПЫТ

**NICOLAS NAZIM ZOUGHEIB**

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***ENHANCING ESTIMATION  
OF EFFICIENCY OF ECONOMIC  
ENTITIES: DATA ENVELOPMENT ANALYSIS***

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The objective of this paper is to describe the Data Envelopment Analysis technique (DEA) used to measure the relative efficiency of decision-making units (DMUs), description of the DEA models and processes, as well as analysis of the technique concerned. The paper shows that DEA plays an important role in measuring relative efficiency and works well even with a small sample of organizations. In accordance with the technique, efficiency and productivity are measured by computing the output to input ratio. The DEA method is used throughout the world by various researchers for evaluating efficiency of different organizations, such as banks, universities and hospitals in different countries.

**Keywords:** efficiency measurement; data envelopment analysis; decision-making unit; production possibility frontier.

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**Introduction.** There are two main methods used in studying efficiency, the first method is the stochastic frontier analysis\*, and the second method is the data envelopment analysis. The stochastic frontier analysis (SFA) is a parametric method used for economic modeling, while the data envelopment analysis (DEA) is a non-parametric approach used for the estimation of production frontier. Jemric and Vujcic (2002) defined DEA as follows: “Data Envelopment Analysis is a nonparametric, deterministic methodology for determining the relatively efficient production frontier, based on the empirical data on chosen inputs and outputs of a number of entities called Decision Making Units” [1]. Charnes, Cooper, and Rhodes introduced data envelopment analysis in 1978\*\* to determine the relative efficiency of operating entities, called decision-making units (DMUs), these mutually comparable entities consume the same inputs and create the same outputs. These units in the context of services can be various service organizations like banks, hospitals, and schools. DEA does not propose a

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*Nicolas Nazim ZOUGHEIB, Ph.D. Candidate, Belarus State Economic University (Minsk, Belarus).*

\* Introduced by Aigner, Lovell and Schmidt [2], Meeusen and Van den Broeck [3].

\*\* Introduced initially by Charnes, Cooper and Rhodes in 1978 [4].

predetermined function form linking inputs and outputs, thus avoiding any model misspecification. DEA works well even with a small sample of organizations; however, DEA's main disadvantage is that it does not account for random errors, and therefore it might overestimate the inefficiency term.

To determine the efficiency of DMUs, an efficient frontier is determined empirically via the observed values of efficient DMUs, and all these efficient DMUs are assigned an efficiency score of one (or 100%). The efficient frontier envelops the set of observations and whenever a DMU lies in the interior of the linear or log-linear envelopment surface, then that DMU is not operating efficiently and, according to the distance from the point representing its input and output values to the corresponding reference point on the efficient frontier, receives an efficiency score greater than zero but less than one (one represents the score of the efficient DMU).

The inefficiency is described as output shortages or input surpluses or both and this inefficiency can be eliminated by reaching an efficient operation point on the efficient frontier. After identifying the entities that will be used as performance benchmarks for inefficiency comparison, the efficient operating point is obtained using the DEA model for each inefficient entity. In addition, DEA doesn't require to designate in advance the weights of each input and output, instead, these weights are determined using the linear programming LP model itself. Moreover, DEA can handle at the same time multiple inputs and outputs to provide a single relative performance efficiency score.

**Methodology and objectives.** Efficiency concept is a major issue that is raised in almost every economy. Defined simply, efficiency is producing more outputs using fewer inputs by performing the required duties with minimum waste of the required resources. In order to improve their efficiency, most firms adopted various economic systems and policy options. This paper introduces an overview of Data Envelopment Analysis, the evaluation process of DEA, the main DEA models and their extensions. The main driving forces for the DEA development are mathematics, economics and management, however, efficiency measurement and optimization represent the fundamental reason for the rapid development of DEA.

The methodology used in this paper is based on secondary data collected from previous studies, journals, and books. To introduce the importance and the growing usage of DEA, the following objectives are set for this research paper:

- describe the DEA models and their extensions;
- discuss the two main processes measured us DEA which are efficiency and productivity;
- introduce the usage of DEA to measure the activities of service institutions such as banks, hospitals and educational organizations.

**Literature review.** A lot of research has been conducted using DEA to measure efficiency and capacity of firms in different sectors, such as finance, health care, and education. During the last decade a large number of international studies of banks' efficiency using DEA were carried out in the European countries and the United States. Sherman and Gold's study (1985) [5] was among the first to apply DEA in the analysis of banks' efficiency in 14 US banks. Recently several studies have been performed concerning the efficiency of banks situated in the Middle East region. Oral and Yolalan (1990) assessed the efficiency of 20 branches of Turkish commercial banks [6]. Al-Faraj et al. (1993) applied DEA to evaluate the relative efficiency of 15 branches of the biggest bank in Saudi Arabia, based on the data for only one year with eight inputs and seven outputs [7]. Ibrahim H Osman, et al. (2008) implemented a DEA method to measure the relative performance of Lebanese banks over the period 1997-2004; this study

made a conclusion that smaller banks tend to transform their inputs into outputs more effectively than larger banks [8].

Several studies using DEA have been conducted in the health care sector. Campanella et al. (2017) based their analysis [9] on 50 hospitals in Italy, they attempted to find out if the hospital's efficiency would improve when these hospitals spend less, maintaining quality. The finding of their study indicated that DEA could be of considerable value for hospital's managers and policy makers who need an explicit and reproducible method to support their decision for resources allocation [9].

Furthermore, Wang and Gao (2017) [10] set an objective to identify the hospital's efficiency determinants and to find ways to improve the performance of Maternal and Child Health Hospitals (MCHH) in terms of their efficiency and productivity. The conclusion was drawn that the county-level MCHHs' overall operational efficiency in Guangxi was found low and the hospitals needed some improvements.

Maree M. Alzghoul et al. (2020) reviewed previous efficiency measurement studies in the health sector that used the DEA model [11] and revealed that DEA is a main tool for measuring the performance of hospitals and some other organizations such as banks. However, this study suggests that the effectiveness of the DEA must be improved and that there is a need to investigate the role of DEA in measuring efficiency of hospitals performance.

**DEA Models.** A variety of DEA models have been developed to measure the capacity and efficiency of firms in many ways. These DEA models are mainly classified as either input-oriented or output-oriented. The input-oriented model uses linear programming equations to determine how to use the firm inputs efficiently in order to achieve the same level of output. In contrast, the output-oriented model configures the linear programming equations to determine the optimal level of output that a firm could achieve using the same inputs, and if this firm operates efficiently along the best practice frontier. There are two commonly used DEA models, the first is the CCR model, named after Charnes et al. (1978) and the BCC model, named after Banker, Charnes, and Cooper (1984), and they yield two different types of envelopment surfaces. The CCR model performs the efficiency measurement under constant returns-to-scale (CRS), which measures inefficiencies according to the size of the operations and the configuration of the selected inputs and outputs. The BCC model assumes variable returns-to-scale (VRS), this model results in a pure technical efficiency score which purely reflects managerial under-performance.

**CCR Model.** In 1978 Charnes-Cooper and Rhodes (CCR) [4] modified the first DEA model developed by Farrel in 1975, which based the efficiency measurement on the production theory using single input and single output:

$$\text{Efficiency ratio} = \frac{\text{Output}}{\text{Input}}.$$

The fact that total efficiency measurement commonly requires the use of multiple inputs to produce single or multiple outputs led to modify the original equation to perform measurement of multiple inputs and multiple outputs. In 1978, Charnes-Cooper and Rhodes CCR developed the basic CCR DEA model by altering the original efficiency equation. The CCR model uses the ratio of weighted outputs to weighted inputs in order to obtain an efficiency score for each decision-making unit under study. The basic equation for CCR DEA model is:

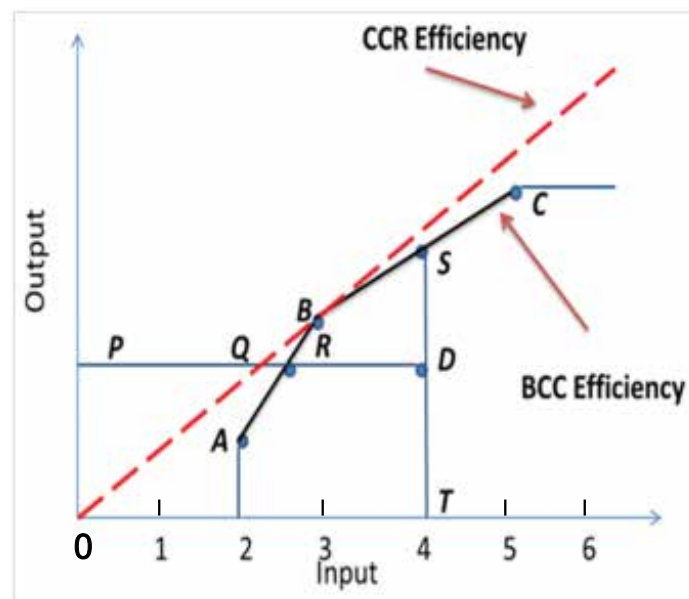
$$DMU \text{ Efficiency Score} = \frac{\text{Weighted Sum of Outputs}}{\text{Weighted Sum of Inputs}}$$

In 1984 Sherman modified the DEA model to measure the performance of banks, and since then DEA was substantially used by banks to measure their operational efficiency. This efficiency measurement is performed using multiple inputs-outputs and within multiple DMUs. When DEA is employed to measure banks' efficiency for a set of DMUs, the linear programming algorithm will calculate the efficiency score of each DMU using the identical inputs and outputs variables, the DMU with the maximum ratio of weighted sum of output to the weighted sum of input is the most efficient DMU, other DMUs will be benchmarked against the most efficient DMU causing the best-practice DMUs to lie on the efficient frontier line. The units with DEA efficiency score of 1 are considered relatively efficient. The non-negativity restrictions ensure that the inputs and outputs will have positive weight values, and that the efficiency score of each unit is between 0 and 1. Units with relatively low productivity will have an efficiency score <1. The degree of inefficiency is computed relative to the frontier, graphically, the production frontier of the CCR Model locates the efficient DMUs on a diagonal line across the area where the frontier and other DMUs lie (production possibility set).

**BCC Model.** The first extension of the basic CCR model is called the DEA BCC model, with other criteria being the same as those in the CCR model, except that it complements the equation to measure input excesses and output shortfalls. The BCC model includes convexity condition with non-negative element constraints.

Basically, in the BCC model, after the efficiency score of each DMUs is calculated, the most efficient DMUs lie on the convex line creating efficient frontier after passing through the area of DMUs (production possibility set).

The following Figure illustrates a comparison of the production frontier of both the Charnes-Cooper-and Rhodes model and Banker-Charnes-Cooper model:



Production frontier of the CCR model and the BCC model  
(Adopted from Cooper et al., 2006)

**Other Models.** In addition to the two basic DEA models (CCR and BCC models) that require differentiation between input-oriented and output-oriented models, the third DEA model is the additive model that combines both orientations into a single model, the additive model considers simultaneously the input excess and the output shortfall to arrive at a point on the efficient frontier. The additive and the extended additive model relate the DEA model to the inefficiency analysis discussed earlier by Charnes-Cooper. The fourth DEA model is the Multiplicative model that provides a log-linear envelopment or a piecewise Cobb-Douglas interpretation for the production process.

The slacks-based measure of efficiency (SBM) augments the additive model. The SBM introduces a measure that makes the efficiency evaluation, as described in the objective, invariant with respect to the measured units. Furthermore, this measure depends on the data of the selected DMUs and not affected by the whole data set. The main difference between SBM and CCR is that the dual variable of the SBM model can be interpreted as profit maximization, while in the CCR model it is interpreted as ratio maximization.

**DEA Process.** DEA models are used to measure the activities of many service institutions such as banks, hospitals and educational organizations. The main two processes measured using DEA are efficiency and productivity, this measurement is performed by computing the output to input ratio. Productivity measurement has two dimensions, the first measures partial productivity and does not consider all output-input factors. The second measures total factor productivity and takes into account all output-input factors. Therefore, total factor productivity measurement avoids assigning mistakenly any gains to one output, while this gain belongs to another output. There exist many challenges when all inputs and outputs are considered, the first is that a single ratio will be used to evaluate the performance of a firm using multiple inputs and outputs. The second challenge is overweight or underweight of an attribute while assigning the weights to each attribute. The third challenge is to handle a large number of variables and constraints.

In the basic DEA models, each DMU is classified as either efficient or inefficient, the efficiency score is used to rank the inefficient entities, while the ranking of the efficient ones under the DEA context is subject to further post-analysis. Adler et al. (2002) [12] grouped the DEA ranking methods into basic areas according to specific criteria, however, recent studies introduced advanced models with their applications, these models are attributed to practical situations. Many models such as CRS, VRS, additive models, and slacks-based measures are reviewed later for developing their methodologies. Their revision introduced different types of variables, such as non-discretionary, non-controllable, categorical and ordinal, they also include undesirable factors and flexible measures, such as sensitivity analysis for problem size issues, data uncertainty and probability-based analysis. Although some developed models went beyond the usual definition of DEA, however, the usefulness and appropriateness of all these DEA-based approaches in the performance measurement have been proved by their increasing usefulness and popularity.

**DEA Usage.** Based on a detailed literature review, numerous studies used Data Envelopment Analysis in their methodology, DEA is widely used for evaluating efficiency and productivity throughout the world by academics and researchers of different organizations operating in such organizations as banks, universities and hospitals. The frontier analysis is considered an effective alternative to traditional bank management tools, it allows management to perform an objective analysis in complex operational environments. Compared to other techniques, Data Envelopment Analysis (DEA) is a good way to organize and analyze data since

it allows efficiency to change over time and it doesn't require an assumption in advance about the specification of the best practice frontier. In addition, DEA allows managers to include random errors into the model if needed.

Following the financial crisis of 2008, banks were forced to restructure their risk management activities and strategies to be able to survive. Banks start to search for more accurate credit scoring models to assess precisely the creditworthiness of their counterparties. These models are designed to accurately detect the ability of these banks to meet the capital adequacy requirements while facing severe economic conditions. The main task of today's risk managers is to look for new methodologies that may lead to better estimation of credit risk and other risks, banks can reduce their losses and overcome crises only if they can create a model that would quickly detect the undesirable situations.

Traditionally, banks have evaluated their performance using many profitability measurement techniques. Ratio analysis was the most frequently used technique that focuses on different aspects of bank operations. However, ratio analysis doesn't relatively provide significant information when the effects of the economies of scale are considered, and when the overall performance measurement of firms is considered. Banks as financial intermediaries facilitate the transfer of financial resources effectively and efficiently for a return. As banks perform this service, depositors and investors are both interested to receive information about bank's performance. That is why there is a need to measure the efficiency of banks, and DEA is the most wide-spread model used to perform such an assessment. Several modifications to the basic CCR-DEA models have been suggested by adjusting the linear programming equations to suit each research.

**Conclusion.** This paper provides a comprehensive investigation of the empirical literature related to the efficiency measurement using DEA. Previous studies were examined to make a historical overview of DEA models, its process, and its usage in performing efficiency measurement analysis in different sectors. The conclusions drawn in this paper will enhance the understanding of policymakers and researchers of the role of DEA in measuring an entity's relative performance efficiency. The selection of the inputs and outputs is essential for assessing the efficiency of decision-making units because the DEA is seen to be a system that is likely to reach a higher performance level without large increase in inputs. In general, DEA plays an important role in measuring efficiency, however, recent studies were concerned with DEA as a tool of measuring performance of some organizations such as hospitals, universities, schools, and banks. It can be concluded that future studies may benefit from this investigation findings regarding the role of DEA in measuring the performance of some organizations, by selecting the most suitable model and the best combination of inputs and outputs.

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## НИКОЛАС НАДЫМ ЗУГЕЙБ

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### СОВЕРШЕНСТВОВАНИЕ ОЦЕНКИ ЭФФЕКТИВНОСТИ СУБЪЕКТОВ ЭКОНОМИЧЕСКОЙ ДЕЯТЕЛЬНОСТИ: АНАЛИЗ ОБОЛОЧКИ ДАННЫХ

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**Об авторе.** *Николас Надым ЗУГЕЙБ, аспирант кафедры математических методов в экономике Белорусского государственного экономического университета (г. Минск, Беларусь).*

Основной целью данной статьи является описание методики анализа оболочки данных (DEA), используемой для измерения относительной эффективности самостоятельных хозяйственных единиц, представление соответствующих моделей и процессов, а также анализ использования разработанной методики. В статье показано, что методика DEA играет важную роль в измерении относительной эффективности и хорошо работает даже с малой выборкой организаций. В соответствии с представленной методикой эффективность и производительность измеряется с помощью отношения выпуска к затратам. Методика DEA широко используется во всем мире разными исследователями для разных организаций, таких как банки, университеты и больницы в разных странах.

**Ключевые слова:** измерение эффективности; анализ оболочки данных; субъект принятия решений; граница производственных возможностей.

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